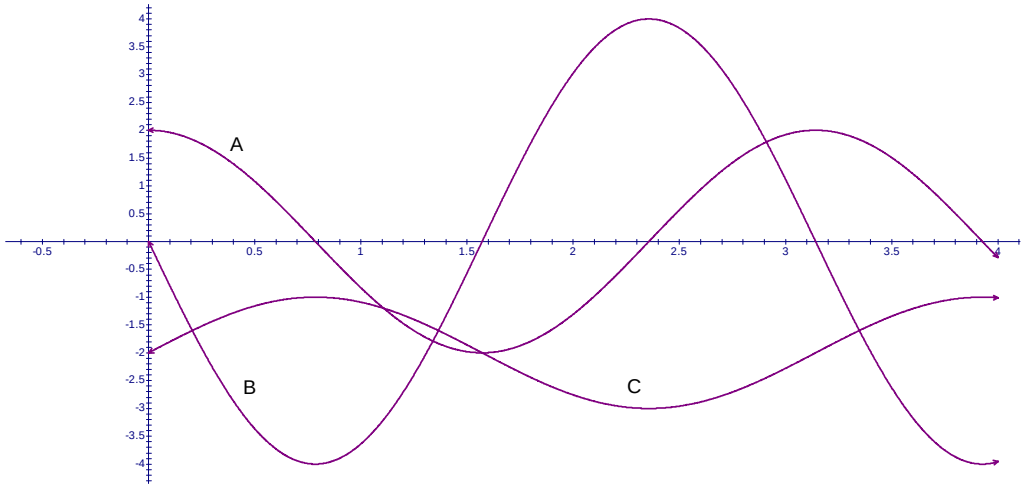
MATH 495.595 / PRACTICE FINAL EXAM QUESTIONS

In addition to reviewing every homework question for the term, make sure you can do each of the following:

1. The graphs A, B and C are a function and its first and second derivatives (not necessarily in that order). Which is which? For which values of x is $f(x)$ increasing, decreasing, concave up, concave down, at a local max or at a local min? What can you say, if anything, about $f'(x)$ and $f''(x)$ on those intervals?



2. Three whole numbers, a, b and c add to 99. The second number is half of the first number. What is the maximum product a × b × c? Use calculus to determine your answer and check your answer using the second derivative (explain).

3. Use algebra and calculus to describe and then graph the function $f(x) = -64x + x^3$ in as much detail as your can.

4. Suppose a rock is hurled vertically up (on Earth). In the following cases:
   (i) Find the complete height above the ground, velocity and acceleration functions
   (ii) Completely describe the path of the rock (max height, when did it hit the ground, how fast was it going when it hit the ground)
   (iii) Determine the speed of the rock at $t = 1.5$ seconds (find the equation for and graph the corresponding tangent line) and find another time when the rock is going the same speed it was going at $t = 1.5$ seconds (find the equation for and graph the corresponding tangent line).
   (iv) Find the average speed of the rock over a three second time interval (what calculus concept does this relate to?).

   a. Rock is 186 feet above ground at $t = 2$ seconds and velocity at $t = 3$ seconds is 4 feet / second.

   b. Rock is 136 feet above ground at $t = 2$ seconds and velocity at $t = 4$ seconds is -78 feet / second.

5. Find the derivative

a. $y=(1+3x^2-x)(x+4x^3)$ b. $y=(1+7x)(3x+4)^{13}$ c. $y=(1+7x^3)^{12}$ d. $y=\dfrac{1+7x}{2-x^4}^3$

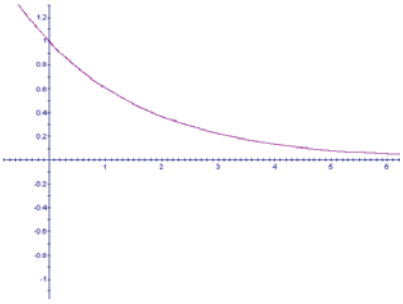
6. Sketch the function with the following piecewise defined components:

$f(x)=-x$	$f(x)=3$	$f(x)=2x-1$	$f(x)=x^2-16$
$-2\le x<0$	$0\le x\le 2$	$2<x<5$	$5<x\le 7$

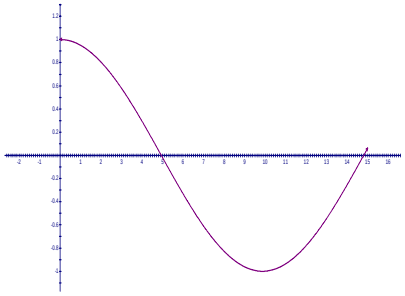
Give the left, right and two-sided limit and function value at all interval endpoints. Additionally explain whether the function is continuous and / or differentiable at each interval endpoint. What is the differentiable / continuous relationship in general? Can you have one, both, neither at a point?

7. For each of the following graphs, i) resketch the graph on graph paper, assume the graph is the function and sketch the derivative, ii) resketch the graph on graph paper, assume the graph is the derivative and sketch two different antiderivatives and iii) resketch the graph on graph paper, assume the graph is the second derivative and sketch the first derivative and the function.

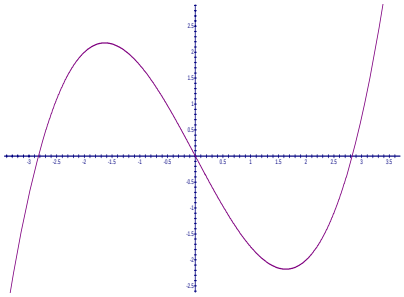
a.



b.



c.



8. Using calculus, compute each of the following:

a. $\int_{-2}^{4} 2x^3+x\ dx$ b. $\int_{1}^{4} \dfrac{2}{x^3}\ dx$ c. $\int_{1}^{3} \sqrt{2x+1}\ dx$:

Note, make sure $F'=f$ where $f=\sqrt{2x+1}$

9. For the function $f(x)=(x-3)\sqrt{2x+6}$
   a. Sketch the graph
   b. Use calculus to determine the minimum value of the function
   c. Use calculus to explain where the function is increasing and where the function is decreasing.

10. Sketch a graph that is continuous, but not differentiable, at $x = 1$ and discontinuous as $x = -3$, but continuous everywhere else.