MATH 495.595 / PRACTICE FINAL EXAM QUESTIONS
SOLUTIONS / COMMENTS

1. C is $f(x)$, A is $f'(x)$ and B is $f''(x)$

2. $a+b+c=99$ $\quad \frac{a}{2}=b$ $\quad a+\frac{a}{2}+c=99 \Rightarrow c=99-\frac{3a}{2}$

Maximize product $a\times b\times c = a\times\frac{a}{2}\times(99-\frac{3a}{2}) = \frac{99a^2}{2}-\frac{3a^3}{4}=P(a)$ $\quad P'(a)=99a-\frac{9a^2}{4}$

$0=99a-\frac{9a^2}{4}\Rightarrow 99a=\frac{9a^2}{4}\Rightarrow 11=\frac{a}{4}\Rightarrow a=44$ Thus, $a=44$, $b=22$ and $c=33$.

3. (Hints) Find x-intercepts, max and min points ($f'(x)=0$,), find where the function is increasing ($f'(x)>0$), where the function is decreasing ($f'(x)<0$), where the function is concave up ($f''(x)>0$), where the function is concave down ($f''(x)<0$).

4. Suppose a rock is hurled vertically up (on Earth). In the following cases:
   a.
   (i) Distance: $d(t)=-16t^2+v_o t+h_o$ feet
   Velocity: $v(t)=-32t+v_o$ feet / second
   Acceleration: $a(t)=-32$ feet / second^2
   Given $d(2)=186$ and $v(3)=4$
   $v(3)=-32(3)+v_o=4\Rightarrow v_o=100\Rightarrow v(t)=-32t+100$
   $d(t)=-16t^2+100t+h_o$
   $d(2)=-16(2)^2+100(2)+h_o=186\Rightarrow h_o=50\Rightarrow d(t)=-16t^2+100t+50$

   (ii) $d(t)=-16t^2+100t+50=0\Rightarrow t\approx 6.71$, At $v(6.71)=-32(6.71)+100\approx -114.7$
   The velocity was -114.7 feet / second which means the ball hit the ground, going down, at a speed of 114.7 feet second at $t\approx 6.71$ seconds.

   $v(t)=-32t+100=0\Rightarrow t=3.125\Rightarrow d(3.125)=206.25$
   The maximum height of the ball is 206.25 feet above ground

   (iii) $v(1.5)=-32(1.5)+100=52$, $v(t)=-32t+100=-52\Rightarrow t=4.75$
   $d(1.5)=164$ $\quad d(4.75)=164$
   $y-164=52(t-1.5)\Rightarrow y=52t+86$. Tangent line for 1.5 seconds
   $y-164=-52(t-4.75)\Rightarrow y=-52t+411$. Tangent line for 4.75 seconds

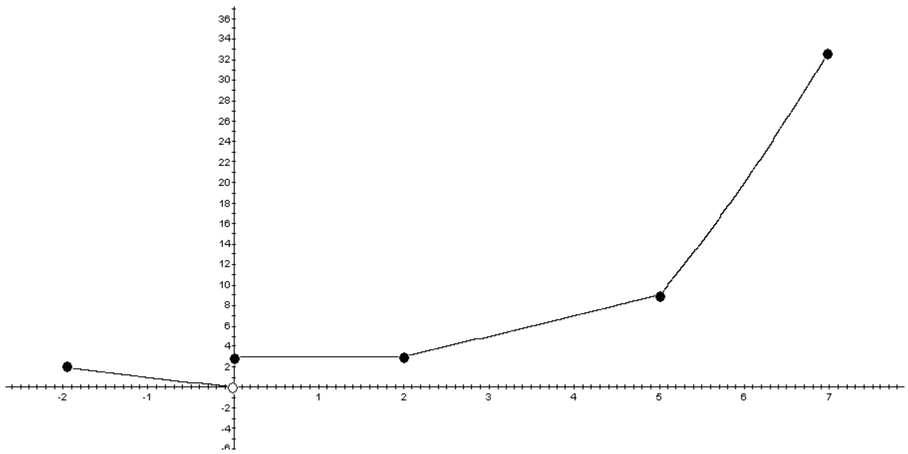
   (iv) Slope of secant line from any three second interval, example:
   $\frac{d(4)-d(1)}{4-1}=\frac{60}{3}=20$ feet / second, average speed of rock from one to four seconds.

   b. Proceed in the same way as for part a).

5.

a. $y' = (1+3x^2 - x)(1+12x^2) + (6x-1)(x+4x^3)$
b. $y' = (1+7x)13(3x+4)^{12}(3) + (7)(3x+4)^{13}$
c. $y' = 12(1+7x^3)^{11}(21x^2)$
d. $y' = \dfrac{(2-x^4)(21x^2) - (1+7x^3)4x^3}{(2-x^4)^2}$

6. Sketch the function with the following piecewise defined components:



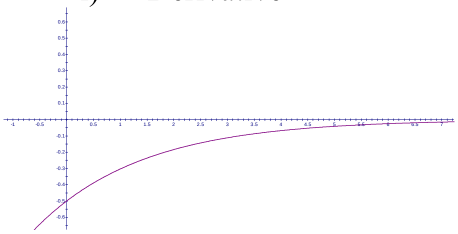
$\lim_{x \to -2+} f(x) = 2$ $\lim_{x \to -2-} f(x)$; endpoint, can't tell	$f(-2) = 2$	$f'(x) = -1$, differentiable and continuous
$\lim_{x \to 0+} f(x) = 3$, $\lim_{x \to 0-} f(x) = 0$, $\lim_{x \to 0} f(x) = DNE$	$f(0) = 3$	Jump, not continuous, not differentiable
$\lim_{x \to 2+} f(x) = 3$, $\lim_{x \to 2-} f(x) = 3$, $\lim_{x \to 2} f(x) = 3$	$f(2) = 3$	Limit same as function value, continuous $f'(2) = 0$, from the left $f'(2) = 2$, from the right "sharp point" not differentiable
$\lim_{x \to 5+} f(x) = 9$, $\lim_{x \to 5-} f(x) = 9$, $\lim_{x \to 5} f(x) = 9$	$f(5) = 9$	Limit same as function value, continuous $f'(5) = 2$, from the left $f'(4) = 10$, from the right "sharp point" not differentiable
$\lim_{x \to 7-} f(x) = 33$ $\lim_{x \to 7+} f(x)$; endpoint, can't tell	$f(7) = 33$	$f'(x) = 2x$, differentiable and continuous

A function can be continuous and not differentiable at a point (sharp corner), but if a function is not continuous at a point, it is also not differentiable.

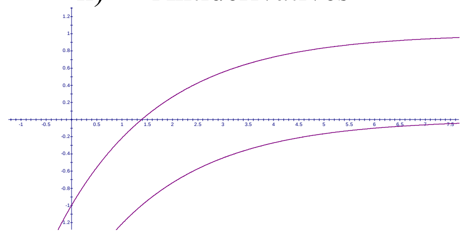
7.

a.

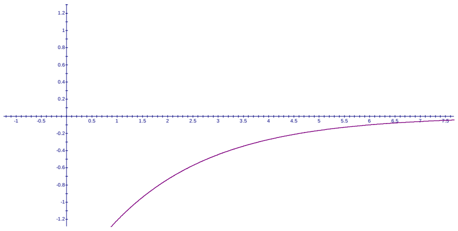
i) Derivative



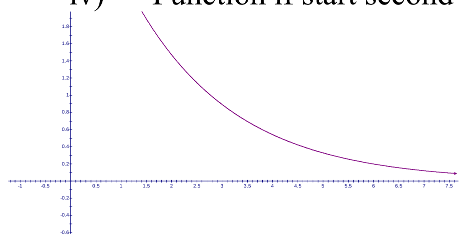
ii) Antiderivatives



iii) Derivative if start second derivative

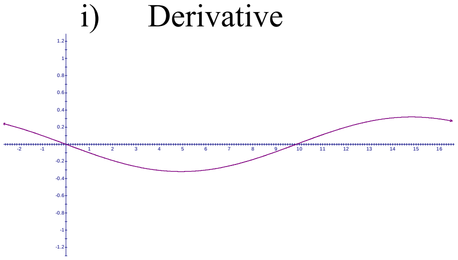


iv) Function if start second derivative

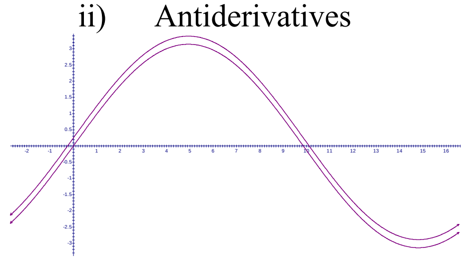


b.

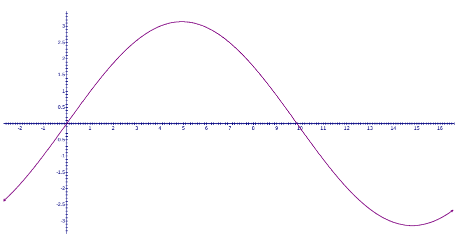
i) Derivative



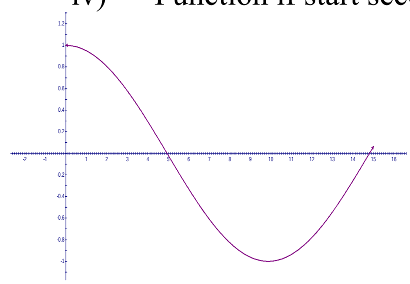
ii) Antiderivatives



iii) Derivative if start second derivative

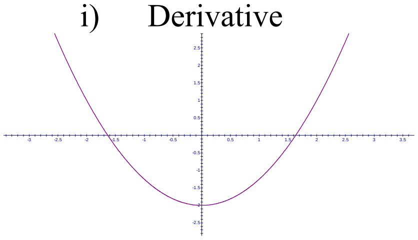


iv) Function if start second derivative

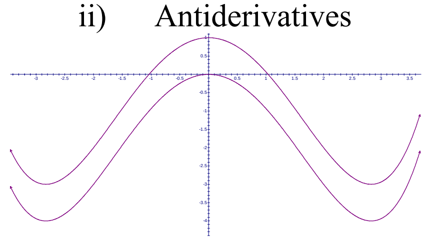


c.

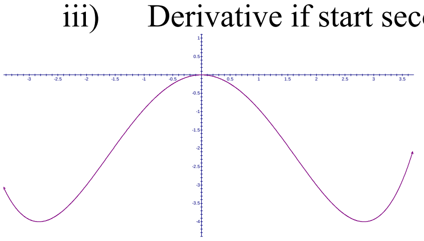
i) Derivative



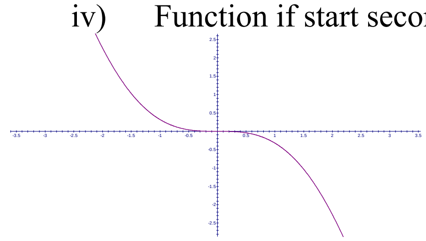
ii) Antiderivatives



iii) Derivative if start second derivative



iv) Function if start second derivative



8. Using calculus, compute each of the following:

a. $\int_{-2}^{4} 2x^3 + x\ dx = \frac{1}{2}x^4 + \frac{1}{2}x^2 \Big]_{-2}^{4} = \frac{1}{2}(4)^4 + \frac{1}{2}(4)^2 - \left(\frac{1}{2}(-2)^4 + \frac{1}{2}(-2)^2\right) = 126$

b. $\int_{1}^{4} \frac{2}{x^3}\ dx = -\frac{1}{x^2}\Big]_{1}^{4} = -\frac{1}{(4)^2} - \left(-\frac{1}{1}\right) = .9375$

c. $\int_{1}^{3} \sqrt{2x+1}\ dx\ \frac{2}{2\times 3}\sqrt{(2x+1)^3}\Big]_{1}^{3} = \frac{\sqrt{(2(3)+1)^3}}{3} - \frac{\sqrt{(2(1)+1)^3}}{3} = 4.44$

9. For the function $f(x) = (x-3)\sqrt{2x+6}$

a. Sketch the graph

Domain for function f(x) is x≥-3, but use calculator to help graph.

b. Use calculus to determine the minimum value of the function.

We want $f'(x) = 0$.

$$f'(x) = (x-3)\frac{1}{\sqrt{2x+6}} + \sqrt{2x+6} = \frac{x-3+2x+6}{\sqrt{2x+6}} = \frac{3x+3}{\sqrt{2x+6}}$$

$f'(x) = 0$ when $x = 1$.

$f(-1) = -8$ is the min (because $f'(x) < 0$ for $x<-1$ and $f'(x) > 0$ for $x>-1$.

c. Use calculus to explain where the function is increasing and where the function is decreasing.

See the answer to part b. (You could use test values $f'(-10)$ and $f'(0)$)

10. Sketch a graph that is continuous, but not differentiable, at $x = 1$ and discontinuous as $x = -3$, but continuous everywhere else.

Draw a smooth function with a cusp (aka "pointy part") at $x = 1$ and a hole (or jump) at $x = -3$.