Topic 15 Homework (Due Tuesday 5/31)

1. $f(x) = -x^2 + 1$

   a. Using an interval of width 0.5, sketch $f(x) = -x^2 + 1$ from $x = -2$ to $x = 3$ with lower bound rectangles (to compute minimum areas) and sketch a second copy of $f(x) = -x^2 + 1$ from $x = -2$ to $x = 3$ with upper bound rectangles (to compute maximum areas). Note, some rectangles will have a height of zero.

   b. Estimate the **minimum,** the **maximum** and the **average** signed areas under $f(x) = -x^2 + 1$ from $x = -2$ to $x = 3$ using function values and your sketches from part a).

   c. How good of an estimate of the signed area from $x = -2$ to $x = 3$ do you think your average estimate is? Is your estimate a little too big or a little too small? Explain.

   d. Use calculus to exactly determine the signed area under $f(x) = -x^2 + 1$ from $x = -2$ to $x = 3$. Use integral notation.

   e. Use integrals and algebra to find a positive value of *a* so that the signed area under $f(x) = -x^2 + 1$ from $x = 0$ to $x = a$ is zero (no credit for guessing).

2. $f(x) = -x(x+3)(x-2)$

   a. Completely multiply out $f(x) = -x(x+3)(x-2)$.

   b. Compute the antiderivative of $f(x)$.

   c. Draw a careful sketch of $f(x)$ and then, for each part, answer the following: Should the signed area be positive or negative?

   (i) From $x = -3$ to $x = 0$?

   (ii) From $x = 0$ to $x = 2$?

   (iii) From $x = 2$ to $x = 4$?

   (iv) From $x = -3$ to $x = 4$?

   d. Using calculus, compute each of the following:

   (i) $\int_{-3}^{0} f(x)\,dx$ (ii) $\int_{0}^{2} f(x)\,dx$

   (iii) $\int_{2}^{4} f(x)\,dx$ (iv) $\int_{-3}^{4} f(x)\,dx$