1. The HW \#1 graph is given on a separate sketch page.

a. The HW \#1 graph is a function and its first and second derivatives. Label the curves $f(x), f^{\prime}(x)$ and $f^{\prime}(x)$ (you don’t need to explain why for this part of the question).
b. For which values of $x$ is $f(x)$ increasing? Decreasing? What can you say about $f^{\prime}(x)$ on those intervals?
c. For which values of $x$ is $f(x)$ concave up? Concave down? What can you say about $f^{\prime}(x)$ on those intervals? What can you say about $f^{\prime}(x)$ on those intervals?
d. For which values of $x$ is $f(x)$ at a non-endpoint local maximum? Local minimum? What can you say about $f^{\prime}(x)$ at those points? What can you say about $f^{\prime \prime}(x)$ at those points?
2. What are the dimensions of a cylindrical cardboard tube that holds 100 cubic inches and uses the least amount of material? Assume the tube and disc ends are made of the same material. Create a Materials function using Total Material $=(2 \times$ circular end area $)+($ tube area $)$. Use calculus to algebraically determine the solution, show your work. Sketch a graph of your materials function and mark the least amount of materials used on the graph. Don't forget units!
3. The function $f(x)=x^{3}+3 x^{2}-72 x$ has one non-endpoint local maximum value and one non-endpoint local minimum value. Assume the domain of the function is all real numbers.
a. Use the first derivative of $f(x)$ to algebraically determine those two local extrema and the second derivative of $f(x)$ to check if the values are local minimums or maximums.
b. Use the second derivative of $f(x)$ to find the inflection points for $f(x)$. Where is $f(x)$ concave up and where is $f(x)$ concave down?
c. Sketch the best possible graph you can for $f(x)$. Use the information you determined in parts a) and b) and needed function values from your calculator table. Determine the x-intercepts for $f(x)$ algebraically, show your work. Label clearly.
4. Suppose a rock is thrown directly upward, and at time $t$ seconds after it is released, the height of the rock is given by the function $h(t)=-16 t^{2}+100 t+120$ feet above ground. Be sure to include units in all of your answers
a. What is the initial height of the rock?
b. What is the initial velocity of the rock?
c. When does the rock hit the ground? Determine this algebraically; show your work.
d. What is the average speed of the rock during the rock's second three seconds of flight? ( $t=3$ to 6 seconds) Determine this algebraically, show your work.
e. What is the speed of the rock when $t=5$ seconds; is the rock going up or down? Use the first derivative to determine this speed algebraically, show your work. At what other time was the rock going the same speed and is the rock going up or down then? Determine this algebraically, show your work. Determine the equations of the tangent lines to the height function for both of these points. Show your work
f. When does the rock reach its maximum speed? Why is this maximum speed? How fast is the rock going?
g. When is the rock at its highest point and how high does it go? Use calculus and determine this algebraically; show your work.
h. What is the acceleration of the rock for any time $t$ seconds?
i. Sketch the height function of the rock, mark all of the points from the preceding questions on the graph. Don't forget to label the axes with units for the height function.

HW \#1 Graph


