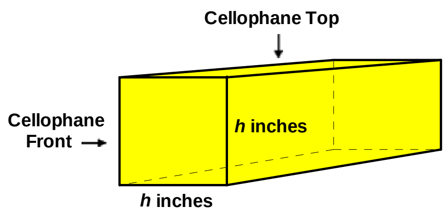
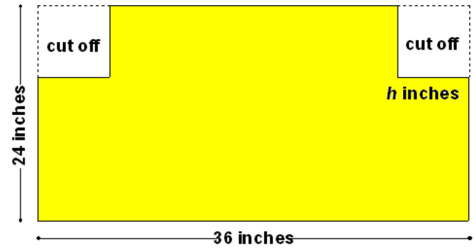
7. *Willy Wonka* wishes to make boxes of maximum volume to display their golden eggs. To construct the boxes, the top and one end will be made of cellophane and the remaining sides will be cut from a sheet of 24 inch × 36 inch gold foil. They plan to cut out two corner squares of length *h* inches from each gold foil sheet as illustrated in the following diagram. The 3-D box diagram shows a square ended box, and the 2-D net diagram would fold to a box with a non-square rectangular end. You may wish to cut and fold a few pieces of scratch paper to get a sense of the box construction (optional).





a. Determine the volume function for the *Willy Wonka* box. Give the volume as a function of the height of the box; $V(h)$ in cubic inches for a height *h* inches.

b. Use your graphing calculator and set the window to show a useful view of the graph of $V(h)$. Sketch the graph of $V(h)$ on graph paper. Label the axes with appropriate numbers and units. Use a reasonable domain and range, list them.

c. Average Rates of Change (show your work for all parts)
   i) Compute the average rate of change of $V(h)$ from $h = 2$ to $h = 5$.
   ii) Compute the average rate of change of $V(h)$ from $h = 5$ to $h = 9$.
   iii) Compute the average rate of change of $V(h)$ from $h = 10$ to $h = 16$.
   iv) Sketch the corresponding secant lines from parts i), ii) and iii) on the graph of $V(h)$ from part b). Label clearly.

d. Which display box configuration will contain the maximum volume? Use the max feature on your calculator to determine the dimensions of the box of maximum volume. Sketch the box and label all dimensions to the nearest tenth of an inch.

e. Instantaneous Rate of Change and Tangent Lines (show your work for all parts)
   i) Numerically approximate the instantaneous rate of change of $V(h)$ at $h = 4$. Determine the equation of the tangent line to $V(h)$ at $h = 4$.
   ii) Numerically approximate the instantaneous rate of change of $V(h)$ at $h = 9$. Determine the equation of the tangent line to $V(h)$ at $h = 9$.
   iii) Numerically approximate the instantaneous rate of change of $V(h)$ at $h = A$. where A is the height of the box with maximum volume (use four decimal places of accuracy for A). Determine the equation of the tangent line to $V(h)$ at $h = A$.
   iv) Sketch the tangent lines from parts i), ii) and iii) on a NEW copy of the graph of $V(h)$.