

Name: _____

MTH 358

Take Home Exam DUE Monday 5/23 by 3p.m.

**YOU ARE ALLOWED TO LOOK AT YOUR BOOK, USE THE COMPUTER,
AND USE YOUR CALCULATOR**

DO NOT CONSULT WITH EACH OTHER!

IF YOU HAVE QUESTIONS, email me at kruczekk@wou.edu

1. Find the general solution to the linear system of differential equations

$$\begin{cases} \frac{dx}{dt} = 2x - \frac{5}{2}y \\ \frac{dy}{dt} = \frac{9}{5}x - y \end{cases}$$

2. Suppose you have a tank that contains 30 gallons of a solution composed of 95% water and 5% Kool-Aid mix. This solution is too weak for you. A second solution containing 75% water and 25% Kool-Aid is added to the tank at the rate of 5 gallons per hour, while at the same time, the entire solution is being drained from the tank (by people drinking it) at the rate of 8 gallons per hour. Assuming the tank is continuously stirred, we want to know the amount of Kool-Aid in the tank over time.

- (a) If $K(t)$ is the amount of Kool-Aid at time t , find a differential equation that models this situation.
- (b) Use Excel and Euler's method (with $h = 0.25$) to approximate $K(t)$ over the appropriate time interval.
- (c) Approximate the percentage of Kool-Aid in the tank right before the tank is empty.

3. Consider the competing squirrel (S) and chipmunk (C) model (where t is measured in years and S and C are measured in 100's of animals):

$$\begin{aligned} \frac{dS}{dt} &= 0.04S - 0.0008S^2 - 0.0008\lambda SC \\ \frac{dC}{dt} &= 0.099C - 0.0006C^2 - 0.0006\lambda SC \end{aligned}$$

- (a) For what values of λ , are there four equilibrium values?
- (b) Suppose people are hunting squirrels (for gravy) and chipmunks (to become performance artists). In particular $H\%$ of each are hunted each year. Modify your system of differential equations to take this into account and find the largest value of H so that there are still four equilibrium values.
4. Fruit Loops is giving away one of five Pirates of the Caribbean toys in each box. I would like to collect 2 Jack Sparrow action figures and one of each of the other four toys. If each toy is equally likely to be in each box, design a simulation to approximate the expected number of boxes of Fruit Loops I need to buy to gather all 6 toys I want (2 Jack Sparrow, 1 Black Pearl, 1 Elizabeth Swan, 1 Will Turner, and 1 Barbossa). (This means you'll need to use Excel to approximate the expected number of boxes of Fruit Loops I need to buy to gather all 6 toys.)

When you are done, any parts that were done on Excel should be emailed to me at kruczekk@wou.edu. However, please put your answers you obtained using Excel on the hard copy of the test you turn in as well. It makes it easier for me to figure out what your answers are this way. Thanks!