MTH 410 - Final Projects

For your final project you will be required to choose one of the following topics, write a short paper, and give a short presentation about it. Your presentation should be a summary of the paper that you have written. Once you have chosen a topic you should consult with me about the details of what you should include in your paper. In general your goal should be to understand the basic definitions, provide a few examples, and prove a few simple properties about your topic. Topics should be chosen by April 29th. You should submit an outline of your paper to me by May 20. Presentations should be about 20 minutes long and will be given during the last week of classes, May 30 - June 3. Final papers are due by midnight on Thursday of finals week, June 9.

1. Connectivity Axioms
   - A space X is connected if for any open subsets $U, V \subset X$ such that $U \cup V = X$ we have that: If $U \cap V = \emptyset$ then either $U = X$ or $V = X$.
   - A space X is path connected if for any two points $x, y \in X$ there is a path $\alpha : [0,1] \to X$ such that $\alpha(0) = x$ and $\alpha(1) = y$.
   - A space X is locally connnected if for every $x \in X$ there exists and open set $U \subset X$ such that $x \in U$ and U is connected.
2. Compactness Axioms
   - A subspace U of a metric space X with metric d is compact if U is closed and bounded.
   - A subspace U of a topological space X is sequentially compact if every sequence in U has a convergent subsequence.
   - A subspace U of a topological space X is compact if for any collection of open sets $\{U_\alpha\}_{\alpha \in \Lambda}$, if $\cup_\alpha U_\alpha = X$, then there exists $U_{\alpha_1}, U_{\alpha_2}, \cdots, U_{\alpha_n}$ for some $n \in \mathbb{Z}$, $n \geq 0$, such that $\cup_{i=1}^n U_i = X$.
3. Separation Axioms
   - A spaces X is T_0 if for any points $x, y \in X$ there is an open set $U \subset X$ such that either $x \in U$ and $y \notin U$, or $y \in U$ and $x \notin U$.
   - A space X is T_1 if for any points $x, y \in X$ there are open sets $U, V \subset X$ such that both $x \in U$, $y \notin U$, and $y \in V$, $x \notin V$.
   - A space X is T_2 or 'Hausdorf' if for any points $x, y \in X$ there are open sets $U, V \subset X$ such that both $x \in U$, $y \in V$, and $U \cap V = \emptyset$.
   - A space X is T_3 if it is T_0 and for any points $x \in X$ and any closed set $F \subset X$ there are open sets $U, V \subset X$ such that both $x \in U$, $F \subset V$, and $U \cap V = \emptyset$.
   - A space X is T_4 if it is T_1 and for any closed sets $F, G \subset X$ there are open sets $U, V \subset X$ such that both $F \subset U$, $G \subset V$, and $U \cap V = \emptyset$.
4. Countability Axioms
   - A space X is second countable if there is a countable basis for its topology.
   - A spaces X is called first countable if at each point $x \in X$ there is a countable basis of neighborhoods of x.

5. Topological Groups
   - A group is a set G along with an associative binary operation $\cdot : G \times G \to G$ such that and identity exists and every element is invertible.
   - A topological group is a group G which is also a topological space such that the map $\cdot : G \times G \to G$ is continuous.
6. Stereographic Projection
   - Let $N, S \in S^2$ represent the north and south pole respectively. There are homeomorphisms $\phi_N : S^2 - \{N\} \to \mathbb{R}^2$ and $\phi_S : S^2 - \{S\} \to \mathbb{R}^2$ called steriographic projections. The restriction of the composition
   $$\phi_S \circ \phi_N| : S^2 - \{N, S\} \to S^2 - \{N, S\}$$
   is a homeomorphism, which proves that S^2 is a manifold.
7. The Homotopy Type of the Letters of the Alphabet
   - Classify all of the letter $A, B, C, \cdots, Z$ up to homotopy.
8. Fixed Point Theorems
   - The Brower fixed point theorem says that any continuous map from the closed unit disk D^n to itself has a fixed point. That is to say that for any continuous function $f : D^n \to D^n$ there exists a point $x \in D^n$ such that $f(x) = x$.
9. Algebraic Varieties and Ideals of Polynomial Rings.
   - For each ideal $I \subset \mathbb{R}[x_1, x_2, \cdots, x_n]$ there is an associated algebraic variety $V(I) \subset \mathbb{R}^n$. Similarly for a subset of $V \subset \mathbb{R}^n$ there is an associated ideal of $I(V) \subset \mathbb{R}[x_1, x_2, \cdots, x_n]$. There is a bijection between prime ideals and closed sets in the Zariski topology on $\mathbb{R}^n$.
10. Category Theory
    - A category is a collection of objects along with sets of morphisms between any two objects.
    - Examples are: Sets with set functions, groups with group homomorphisms, topological spaces with continuous maps, etc.
    - Morphisms between categories are called functors.
11. The Hopf Fibration (This topic is already taken by someone)
    - There is a map $h : S^3 \to S^2$ such that fore every $x \in S^2$ the set $f^{-1}(x) \cong S^1$. This is called the Hopf fibration and it has many interesting properties.
12. Minkowski Space Time (This topic is already taken by someone)
    - The Lorentzian metric is not a metric, but instead a pseudometric of type $(3, 1)$. The space $\mathbb{R}^4$ with this pseudometric is called Minkowski space and is a local model for the space-time continuum.
13. Knot Theory (This topic is already taken by someone)
    - A knot is an embedding of S^1 into $\mathbb{R}^3$.
    - Two knots are equivalent if one can be transformed into the other by a deformation of $\mathbb{R}^3$.