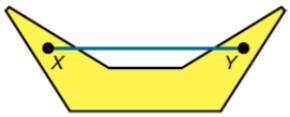
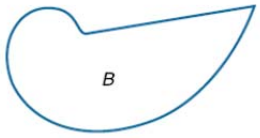
SOLUTIONS **MATH 213 / Final Review Questions, F15**

1. Sketch a **concave polygon** and explain why it is both **concave and a polygon.**
A polygon is a simple closed curve that is the union of line segments.

   A polygon is concave if it contains two points such that the line segment joining the points does not completely lie in the polygon.



2. Sketch a **simple closed curve** and explain why it is both **simple** and **closed**.
A simple closed curve is a simple curve (starts and stops without intersecting itself) that starts and stops at the same point.

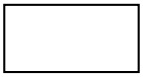


Simple closed curve

3. Determine the measure of the **vertex, central and exterior angles** of a **regular heptagon, octagon and nonagon**.

Shape	n	Vertex $\frac{(n-2)180°}{n}$	Central $\frac{360°}{n}$	Exterior $180° - \frac{(n-2)180°}{n} = \frac{180n° - 180n° + 360°}{n} = \frac{360°}{n}$
regular heptagon	7	128.6°	51.4°	51.4°
regular octagon	8	135.0°	45.0°	45.0°
regular nonagon	9	140.0°	40.0°	40.0°

4. Determine whether each figure is a regular polygon. If it is, explain why. If it is not, state which condition it dos not satisfy.



No, sides are not congruent

No, angels are not congruent

Yes, all angles are congruent and all angles are congruent

5. A prism has 69 edges. How many vertices and faces does it have?
E = 69, V = 46, F= 25

6. A pyramid has 68 edges. How many vertices and faces does it have?
E = 68, V = 35, F= 35

7. A prism has 24 faces. How many edges and vertices does it have?
E = 66, V = 44, F= 24

8. A pyramid has 24 faces. How many edges and vertices does it have?
E = 46, V = 24, F= 24

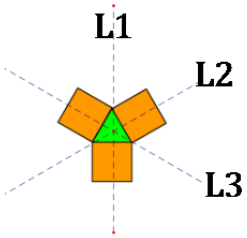
9. Which regular polygons tessellate? Explain and illustrate your answer with a sketch.

Only the triangle, square and hexagon will tessellate since they are the only regular polygons whose vertex angle divides evenly into 360°

10. Sketch a pattern block figure with exactly four rotational symmetries and no lines of symmetry. List the angles of rotation.

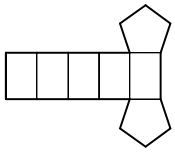
90°, 189°, 270°, 360°
Other figures also work

11. Sketch a pattern block figure that is not a regular polygon, with exactly three lines of symmetry. Mark the lines and label them L1, L2 and L3

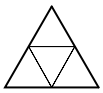


Other figures also work

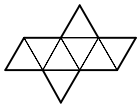
12. What are these nets for? Be exact:



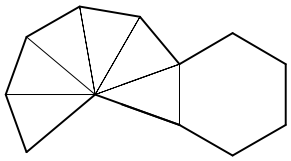
Right, regular pentagonal prism



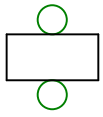
Tetrahedron (Right, equilateral pyramid)



Octahedron

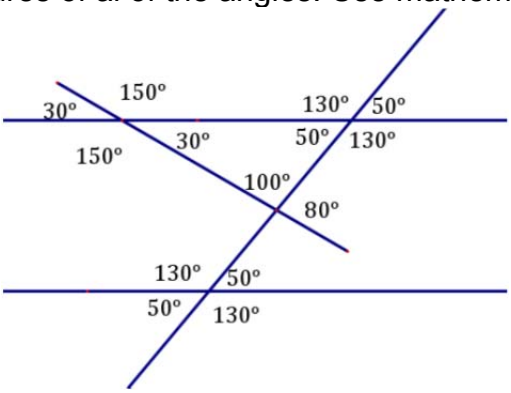


Right, regular, hexagonal pyramid



Right cylinder

13. In the following figure, line l is parallel to line m. Given the angle measures indicated on the figure, find the measures of al of the angles. Use mathematics, not a protractor.



Use congruent vertical angles and congruent alternate interior angles

14. If you halve the length of each side of a rectangle, by what factor does the PERIMETER of the rectangle expand or contract? What if you alter the length by a factor of A?

$\frac{1}{2}$ contracts; A expands if $A>1$, A contracts if $A<1$

15. If you double the length of each side of a rectangular prism, by what factor does the SURFACE AREA of the prism expand or contract? What if you alter the length by a factor of A?

2^2 expands; A^2 expands if $A>1$, A^2 contracts if $A<1$

16. If you double the radius of a sphere, by what factor does the VOLUME of the sphere expand or contract? What if you alter the length of the radius by a factor of A?

2^3 expands; A^3 expands if $A>1$, A^3 contracts if $A<1$

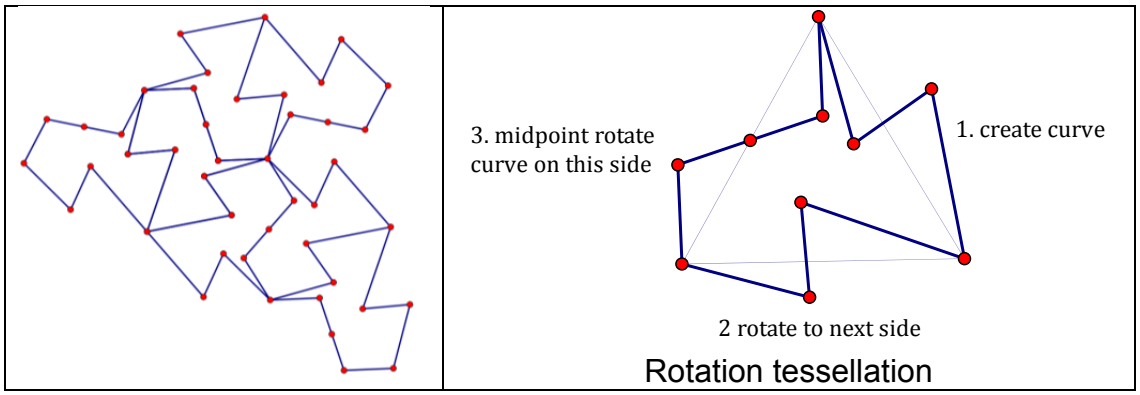
17. How many CUBIC FEET of gravel are needed to cover a 2 yard x 3 yard area filled to 0.9 yard high?

$$2\times3\times.9\ yd^3\times\left(\frac{3\ ft}{yd}\right)^3=2\times3\times.9\times27\ ft^3=145.8\ \ ft^3$$

18. If five tennis balls of diameter 6.5 cm. are packed into a cylindrical can that exactly holds the five balls (the can also has diameter 6.5 cm.); what is the volume of the UNUSED SPACE in the can?

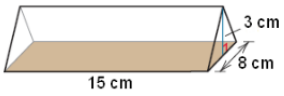
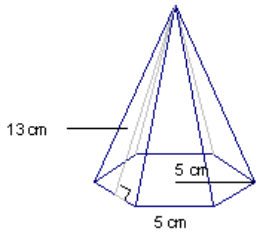
$$\left(\pi\times\left(\frac{6.5}{2}\right)^2\times(5\times6.5)\right)-\left(5\times\frac{4}{3}\pi\times\left(\frac{6.5}{2}\right)^3\right)\approx359.5\ \text{cm}^3$$

19. Modify all three sides of an equilateral triangle to create a shape that tessellates. Briefly explain your work and use tracing paper to aid you as you sketch the beginning of a tessellation using this template. What type of tessellation is it?

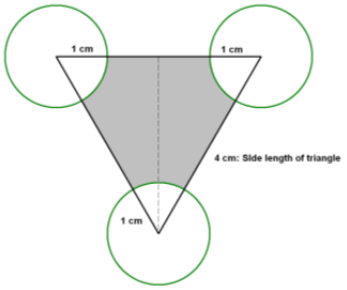


Rotation tessellation

20. Find the surface area and volume of the following objects.

	Surface Area	Volume
	$\frac{1}{2}\times 8\times 3\times 2 = 24$ bases $3^2+4^2=5^2$ $15\times 5\times 2 = 150$ front/back faces $15\times 8 = 120$ bottom face $24+150+120=294$ $\boxed{294\ cm^2}$	$\frac{1}{2}\times 8\times 3\times 15 = \boxed{180\ cm^3}$
	$2.5^2+h^2=5^2 \quad h\approx 4.3$ $\frac{1}{2}\times 5\times 4.3\times 6 \approx 64.95$ hexagonal base $\frac{1}{2}\times 5\times 13\times 6 = 195$ faces $64.59+195=259.95$ $\boxed{259.95\ cm^2}$	$4.3^2+H^2=13^2 \quad H\approx 12.3$ $\frac{64.59\times 12.3}{3} = 264.82 \approx \boxed{264.8\ cm^3}$

21. The following equilateral triangle has three circular regions surrounding it. What is the area of the shaded portion of the diagram?

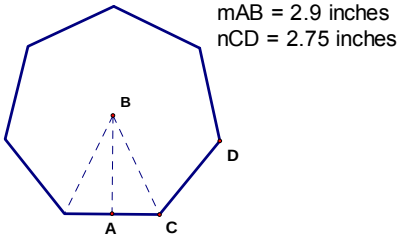


$2^2+h^2=4^2 \quad h=2\sqrt{3}\approx 3.46$

$\frac{1}{2}\times 4\times 3.46 \approx 6.93\ cm^2$ triangle

$3\times\frac{\pi\times 1^2}{6}=\frac{\pi}{2}\ cm^2$ three 60° portions make 1/2 circle

$6.93-\frac{\pi}{2}$ $\boxed{\approx 5.34\ cm^2}$



$\frac{1}{2}\times 2.75\times 2.9\times 7 = 27.9125$

$\boxed{27.9125\ in^2}$

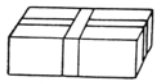
22. What is the area of the regular heptagon pictured above? Show your work. Include units.

23. Lisa is buying special hardwood flooring from Germany and the cost (in US dollars) is $1500 per bundle. Each bundle will cover 12 square meters. If Lisa wants to put the flooring in a room that is 18 feet by 20 feet, how much will the flooring cost? Note: Lisa can only buy whole bundles (e.g. she can't buy 1/2 of a bundle, etc). Show all work and conversions. Note 1 yard ≈ .915 meters.

$18'\times 20'\times\frac{1\ yd}{3\ ft}\times\frac{1\ yd}{3\ ft}\times\frac{.915\ m}{yd}\times\frac{.915\ m}{yd} = 33.489\ m^2$

$33.489\ m^2\times\frac{1\ \text{bundle}}{12\ m^2} = 2.79$ bundles Lisa must by 3 bundles for $4500

24. This package is 12 inches longer than it is wide. It is also 6 inches wider than it is high. If the total length of the ribbon needed to go around both ways is 96 inches long, what is the length of the package?



$l = 12 + w \qquad w = 6 + h$

Ribbon covers $l + l + w + w + 4h = 2l + 2w + 4h$

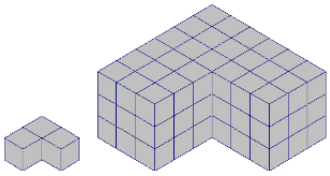
$2l + 2w + 4h = 2(12 + w) + 2w + 4h = 24 + 4w + 4h$

$24 + 4(6 + h) + 4h = 24 + 24 + 8h = 96$

$8h = 48 \quad h = 6$ inches, $w = 12$ inches, $l = 24$ inches

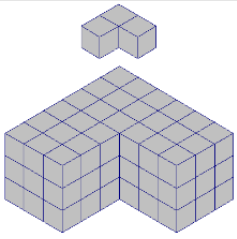
25. The following two figures are similar. Each dimension of the larger figure is TRIPLE the corresponding dimension of the smaller figure. If this tripling of dimensions is continued, how many cubes will there be in the **FOURTH (nth)** figure? If the dimensions of the first figure are doubled rather than tripled, how many cubes would there be in the **SECOND (nth)** figure?

	Tripling dimensions sequence	**Doubling dimensions sequence**
Figure	**Volume**	**Volume**
1	3	3
2	$3 \times 3^3 = 3 \times 27 = 81$	$3 \times 2^3 = 3 \times 8 = 24$
3	$81 \times 27 = 3 \times 27^2 = 2187$	$24 \times 8 = 3 \times 8^2 = 192$
4	$2187 \times 27 = 3 \times 27^3 = 59{,}049$	$192 \times 8 = 3 \times 8^3 = 1536$
n	$3 \times 27^{n-1}$	$3 \times 8^{n-1}$



Alternate solution

	Tripling dimensions sequence	**Doubling dimensions sequence**
Figure	**Volume**	**Volume**
1	3	3
2	$3 \times 3^3 = 81$	$3 \times 2^3 = 24$
3	$(3 \times 3^3) \times 3^3 = 3 \times 3^6 = 2187$	$(3 \times 2^3) \times 2^3 = 3 \times 2^6 = 192$
4	$(3 \times 3^6) \times 3^3 = 3 \times 3^9 = 59{,}049$	$(3 \times 2^6) \times 2^3 = 3 \times 2^9 = 1536$
n	$3 \times 3^{3n-3}$	$3 \times 2^{3n-3}$



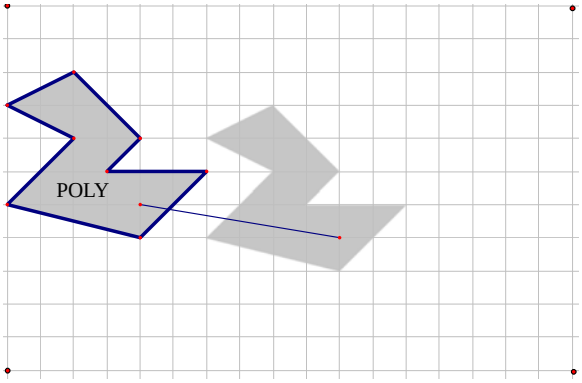
Note that $3 \times 27^{n-1} = 3 \times 3^{3n-3}$ and $3 \times 8^{n-1} = 3 \times 2^{3n-3}$ so both patterns are equivalent

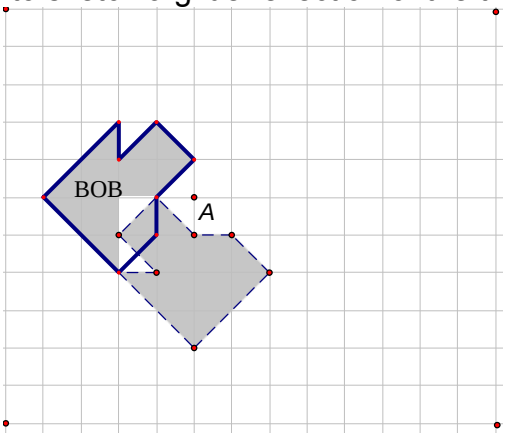
26. Julia and James just bought their first house. They are so excited they had a scale model of their new house made. The scale factor relating their model to the actual house is 75. If the length of one side of the actual house is 50 feet, how many **inches** long is the corresponding side of the model house? If the volume of the model of the house is 0.10 feet^3, what is the volume of the actual house?

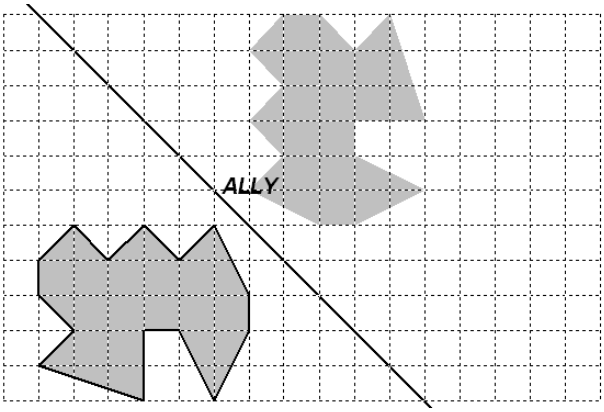
$$50 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{1}{75} = \boxed{8 \text{ inches}}$$

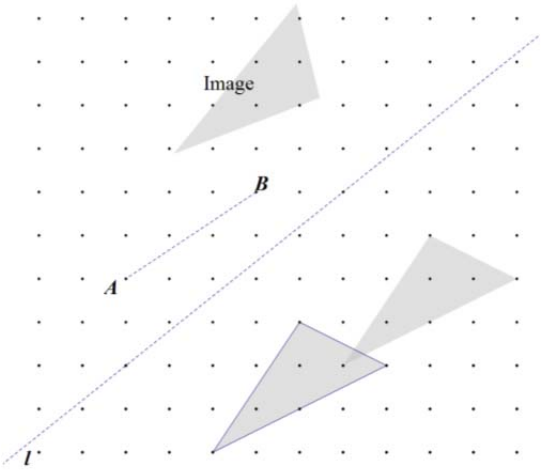
$$.1 \text{ feet}^3 \times \left(\frac{12 \text{ inches}}{1 \text{ foot}} \right)^3 \times 75^3 = \boxed{72{,}900{,}000 \text{ inches}^3}$$

27. Sketch the translated image of POLY for the mapping that maps A to B.
28. Sketch the rotated image of BOB for a 90◦ counterclockwise rotation about point A:
29. Sketch the reflected image of ALLY over the line L
30. Use the motion; point A to point B and line L to sketch a glide reflection of the triangle/

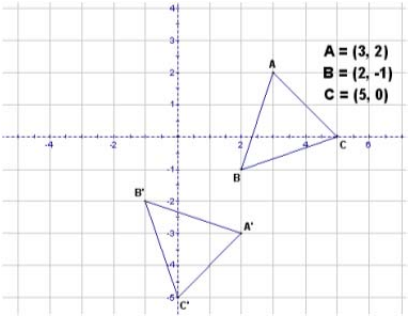








31. In the diagram below triangle A′B′C′ is the image of triangle ABC under a rotation through 90° clockwise centered at the origin.



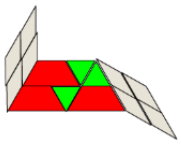
What are the coordinates of A′, B′ and C′?

A′ = (2, -3), B′ = (-1, -2) and C′ = (0, -5)

Generalize the rule for this rotation, i.e., create a rule that would move any point the same way.

(x, y) is moved to the point (y, -x)

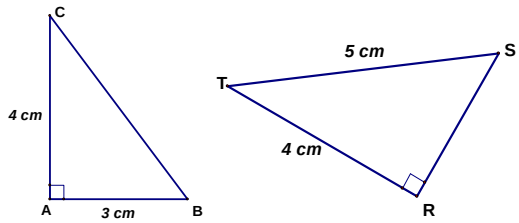
32. The following figure is made using two tan parallelograms and one red trapezoid from the pattern blocks. Sketch an enlargement of the figure by a scale factor of 2. Use as many tan parallelograms and red trapezoids in your enlargement as possible. If instead you were to sketch an enlargement by a scale factor of 3 (or 4 or 5), how many tan parallelograms and how many red trapezoids (or combined blocks of the same area) would be in your figure?

N	Image (2 only)	Parallelograms	Trapezoid (equivalent to)
2		2×2^2	2^2
3		2×3^2	3^2
4		2×4^2	4^2
5		2×5^2	5^2

33. Match the triangles into CONGRUENT or (only) SIMILAR pairs. In each case, solve for unknown values in the triangles (only as needed) and use triangle properties to show why the matched triangle pair is CONGRUENT or SIMILAR. Explain as needed. Show your work. Triangles are not to scale.

$\Delta ABC \cong \Delta RST$ *SSS*

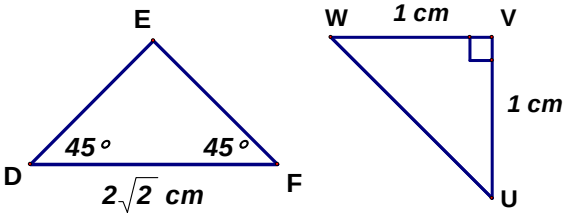
$TS \cong CB$ $TR \cong CA$ $RS \cong AB$ Pythagorean Thm



$\Delta ABC \cong \Delta RST$ by SSS or SAS or ASA

$DE \cong DF \cong WV \cong UV$ and $DF \cong WU$ Pythagorean theorem

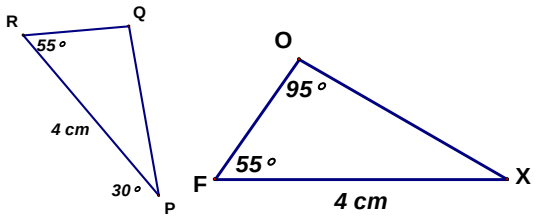
$\angle W \cong \angle U \cong \angle D \cong \angle F$ isosceles right triangles



$\Delta PQR \cong \Delta XOF$ ASA

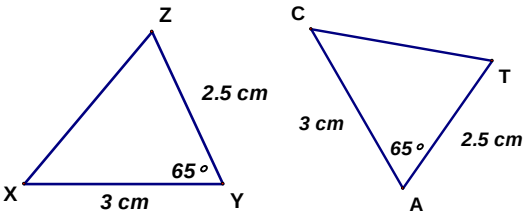
$\angle R \cong \angle F$ $\angle P \cong \angle X$ $\angle Q \cong \angle O$ $RP \cong FX$

Triangle sum angles

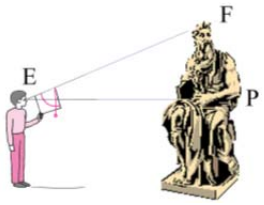


$\Delta ABC \cong \Delta RST$ *ASA*

$XY \cong CA$ $ZY \cong TA$ $\angle Y \cong \angle A$



34. In the following diagram a clinometer was used to determine that the angle of elevation, angle PEF, from the observers eye to the top of the statue is 23°. This knowledge enables us to correctly draw the scale model of triangle PEF which is similar to the actual triangle formed by the observer's eye and the two parts of the statue as shown in the picture. In our scale triangle, the length of side EP is 9 inches and the length of side EF is 10.82 inches. Suppose both the observer and the statue are at ground level, the observer is 40 feet from the statue and the observer's eye is 5 feet above ground. What is the height of the statue?



$$\frac{10.82}{9}=\frac{EF}{EP}=\frac{E'F'}{E'P'}=\frac{E'F'}{40}$$

$$\frac{10.82}{9}=\frac{E'F'}{40} \qquad E'F'=\frac{10.82}{9}\times 40\approx 48.1$$

$48.1+5=53.1$ The statue is about 53 feet high

35. Every equilateral triangle is isosceles.
True by definition

36. A rhombus is a regular polygon.
False, angles are not congruent

37. Every hexagon is a regular polygon.
False, draw a non regular hexagon

38. The acute angles of a right triangle are complementary.
True, angle sum: $A+B+C=180°$ $C=90°$ $A+B=90°$

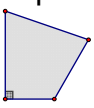
39. The acute angles of a right triangle are supplementary.
False, see 39

40. If two angles are supplementary, one must be either right or obtuse.
True, supplementary means they add to 180°. If they are both < 90°, they can't add to 180°.

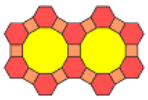
41. Every prism has a square base.
False, consider your textbook which is a rectangular prism with a non-square rectangular base.

42. A quadrilateral may have all acute angles.
False, the sum of the angels is 180°(4 – 2) = 360°. If a + b + c + d = 360° and each one is < 90°, they can't add up to 360°

43. A quadrilateral may have both a right angle and an obtuse angle.

True 

44. A semi-regular tessellation can be made using only regular hexagons and squares.
False, this is 23b from section 9.2



45. Every prism has rectangular lateral faces.
False, if the prism is oblique, the faces are non-rectangular parallelograms

46. Some scalene triangles are right triangles.
True, for example a 3-4-5 right triangle.

47. Every pyramid has isosceles triangles sides.
False, if the pyramid is oblique, the faces are scalene triangles.

48. All quadrilaterals have at least two lines of symmetry.
False, see #44

49. The top of a cone is called a vertex or apex.
True by definition

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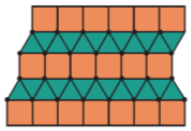
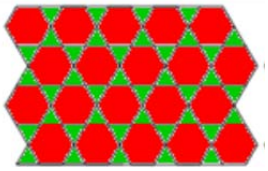
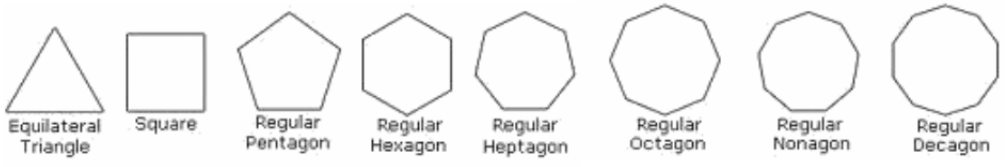
50. Any two regular polygons that tessellate by themselves can be used together to create a semi-regular tessellation.

Square / triangle	Square / hexagon	Hexagon / triangle
 There are other arrangements too	Doesn't work. You can't find a combination of 90° and 120° that adds up to 360° So the answer is **FALSE**	

51. If a polygon is concave, then it must have a reflex angle.
True, in fact this is an alternate definition for concave. Draw any concave polygon to see for yourself.

52. All regular polygons are convex.
True, as the number of sides increase, the polygons start looking more and more circular.



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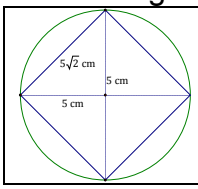
53. 4 quarts = 128 ounces
True $4\ qts \times \frac{4\ cups}{qt} \times \frac{8\ ounces}{cup} = 4 \times 4 \times 8\ ounces = 128\ ounces$

54. If the area of a circle doubles, so does the radius.
False, if the area doubles, the scale factor is $\sqrt{2}$ so the radius, which is a length, goes up by the scale factor.

55. If the surface area of two cylinders is the same, then the cylinders have the same volume.
False, roll a piece of paper the short way and the long way to see this.

56. A cube of with side length 10cm would fit inside of a circle with diameter 10cm.



Not even close, the cube would have to pass through as this square section illustrates and the longest side that could fit would be $5\sqrt{2}\ cm < 10\ cm$

57. 25° C is warmer than 50° F.

$212°F = 100°C \qquad 32°F = 0°C$

True $\dfrac{212\text{-}32}{100} = 1.8 \quad F = 1.8C + b \quad 32 = 1.8°(0) + b \quad b = 32$

$F = 1.8C + 32 \qquad F = 1.8(25) + 32 = 77°,\ 25°C$ is warmer

58. The volume of a right pyramid is 3 times the volume of a right prism with the same base and height.
True

$\text{Volume pyramid} = \dfrac{1}{3}Ah \quad A = \text{area of base} \quad h = \text{height} \quad \text{Volume prism} = Ah \quad A = \text{area of base} \quad h = \text{height}$

59. The diagonals of a kite intersect at right angles.
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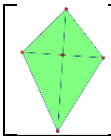

True, draw some kites to see this

60. All right triangles are similar.
False, for example, there are many isosceles right triangles and these are not similar to a 3-4-5 right triangle

61. Triangle 1 has side lengths 2 cm, 3 cm and 5 cm. Triangle 2 has side lengths 2 cm, 3 cm and 5 cm. Triangle 1 and Triangle 2 must be similar.
True, they are congruent by SSS so there are also similar.

62. If a rectangular prism is scaled by a factor of 3, the surface area increases by a factor of 6.
False, increases by 3 x 3 = 9

63. Triangle 1 has angles 40°, 40° and 100° and Triangle 2 has angles 40°, 100° and 40°. Triangle 1 and Triangle 2 must be congruent.
True by AA similarity