Name:

Quiz 3

Read all directions carefully. You must show all work to receive credit. No notes, book, calculators, mp3 players or phones are allowed during this quiz. Write clearly and make sure to indicate your final answer.

1. For the quadratic functions $f(x) = -2x^2 + 4x - 11$

   (a) What is the leading coefficient?
   -2

   (b) Does the graph of $f(x)$ open upward or downward?
   The leading coefficient is negative, $a < 0$, so the graph opens downward.

   (c) Is the vertex of $f(x)$ a maximum or a minimum?
   Because the graph opens downward, the vertex is the highest point on the graph, so it is a maximum.

   (d) Find the coordinates of the vertex of $f(x)$.
   The vertex is at the point
   $$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right).$$
   In this case $a = -2$ and $b = 4$, so the x-coordinate is $\frac{-4}{2(-2)} = 1$. Thus, the y-coordinate is $f(1) = -2(1) + 4(1) - 11 = -9$. Therefore, the vertex is at the point
   $$(1, -9).$$

2. Preform the following arithmetic of complex numbers. Make sure to simplify.

   (a) $(1+2i)(2-3i) = 1(2) + 1(-3i) + (2i)2 + (2i)(-3i) = 2 - 3i + 4i - 6i^2 = 2 + i - 6(-1) = 8 + i.$

   (b) $-i(1+5i) = (-i)1 + (-i)(5i) = -i - 5i^2 = -i - 5(-1) = 5 - i.$

   (c) $i^7 = i^6 i = (i^2)^3 i = (-1)^3 i = (-1)i = -i.$

3. Consider the quadratic function $g(x) = 9x^2 + 3x + \frac{5}{4}$

   (a) What is the discriminant of $g(x)$?
   The discriminant is given by the equation $\Delta = b^2 - 4ac$. In this case $a = 9$, $b = 3$ and $c = \frac{5}{4}$. Therefore, we get

$$\Delta = 3^2 - 4(9)\frac{5}{4} = 9 - \frac{4(9)5}{4} = 9 - (9)5 = 9 - 45 = -36.$$

   (b) How many real roots does $g(x)$ have?
   The discriminant is less than zero, $\Delta < 0$, so the function $g(x)$ has 0 real roots.

   (c) Use the quadratic equation to find all of the roots of $g(x)$.
   The quadratic equation says that the roots of the quadratic function $g(x)$ occur at the values

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

   In this case, $a = 9$, $b = 3$, and $c = \frac{5}{4}$. We have already computed $b^2 - 2ac$ in part 3a. So we get

$$x = \frac{-3 \pm \sqrt{-36}}{2(8)} = \frac{-3 \pm 6i}{18} = \frac{-1 \pm 2i}{6}.$$

   Thus the roots of $g(x)$ (in other words, the solutions to the equation $0 = 9x^2 + 3x + \frac{5}{4}$) are the complex numbers:

$$-\frac{1}{6} + \frac{1}{3}i,$$

   and

$$-\frac{1}{6} - \frac{1}{3}i.$$