Activity Set **9.4**

CREATING SYMMETRIC FIGURES: PATTERN BLOCKS AND PAPER FOLDING

Virtual Manipulatives

www.mhhe.com/bbn

PURPOSE

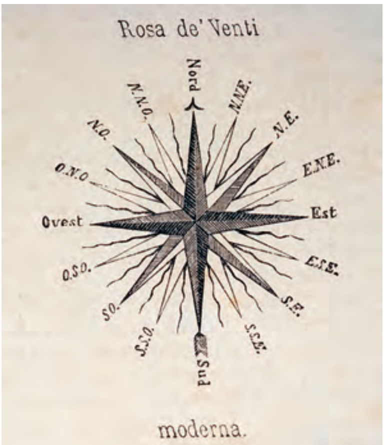
To introduce investigations of symmetry.

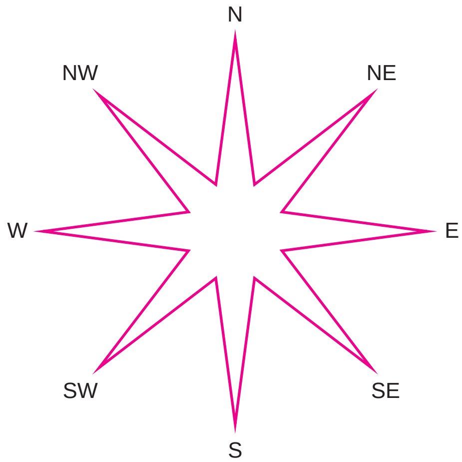
MATERIALS

Pattern Blocks from the Manipulative Kit or the Virtual Manipulatives, Material Card 33, scissors and pieces of blank or scratch paper.

INTRODUCTION

The *wind rose,* shown below on the left, is a mariner's device for charting the direction of the wind. The earliest known wind rose appeared on the ancient sailing charts of the Mediterranean pilots, who charted eight principal winds. These are marked Nord, NE, EST, SE, Sud, SO, Ovest, and NO on the wind rose. Later, half-winds led to a wind rose with 16 points, and quarter-winds brought the total number of points to 32.†





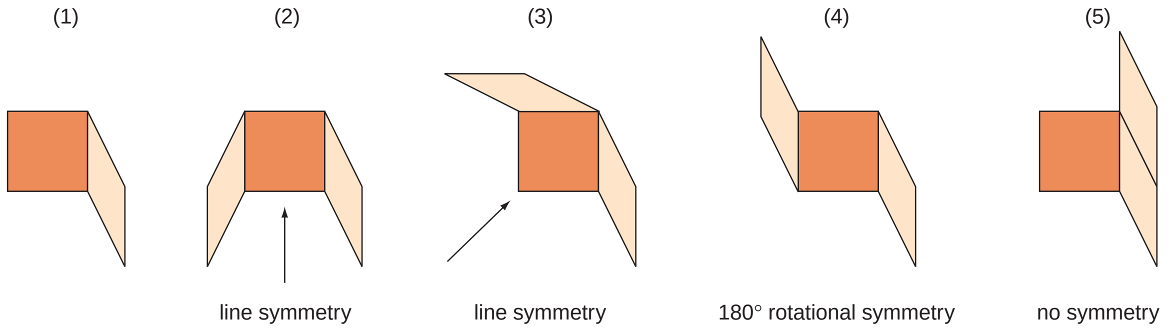
The eight-pointed wind rose shown above at the right is highly symmetric. It has eight lines of symmetry and eight rotational symmetries. For example, a line through any two opposite vertices of the wind rose is a **line of symmetry** because when the wind rose is folded on this line, the two sides will coincide. When a figure has a line of symmetry, the figure is also said to have *reflection symmetry.* The wind rose has **rotational symmetries** of 45°, 90°, 135°, 180°, 225°, 270°, 315°, and 360° because, when rotated about its center through any one of these angles, the figure will coincide with the original position of the wind rose.

In activities 1 and 2 you will be using pattern blocks to construct symmetric figures, and activities 3 to 6 have several basic patterns for cutting out symmetric figures. The variety of figures that can be obtained from slight changes in the angles of the cuts is surprising. The eight-pointed wind rose can be cut from one of these patterns. As you cut out these figures try to predict which one will produce the eight-pointed wind rose.

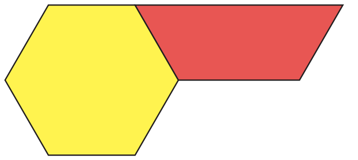
†M. C. Krause, "Wind Rose, the Beautiful Circle," *Arithmetic Teacher,* p. 375.

Pattern Block Symmetries

1. Use the square and the tan rhombus from your pattern block pieces to form figure 1. Notice that when another rhombus is attached as in figures 2 and 3, the new figures each have one line of symmetry. When the rhombus is attached as in figure 4, there are no lines of symmetry but there is rotational symmetry of 180°. Finally, when the rhombus is attached as in figure 5, there are no lines of symmetry or rotational symmetries (other than 360°).



***a.** Form this figure with your pattern block pieces. Determine the different ways you can attach exactly one more trapezoid to this figure to create figures with reflection symmetry. Use your pattern blocks to form at least four of these figures and sketch your results. Mark the lines of symmetry on your sketches.



b. Do the same as in part a but find two ways to create figures with rotational symmetry (other than 360°) by attaching exactly one more trapezoid to the figure above. List the rotational symmetries for each figure.

***c.** By attaching 6 trapezoids to one hexagon, create the following four figures that have the given properties. Sketch your results.

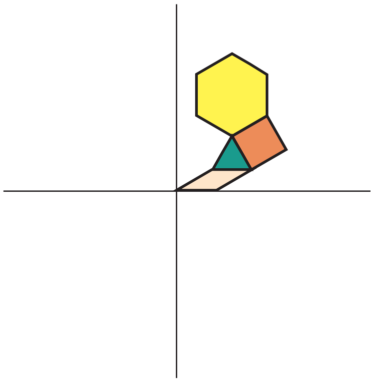
Figure 1: One line of symmetry and no rotational symmetries (other than 360°)

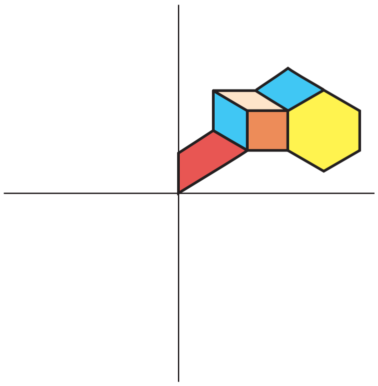
Figure 2: Six rotational symmetries but no lines of symmetry

Figure 3: Three rotational symmetries and three lines of symmetry

Figure 4: No rotational symmetries (other than 360°) and no lines of symmetry

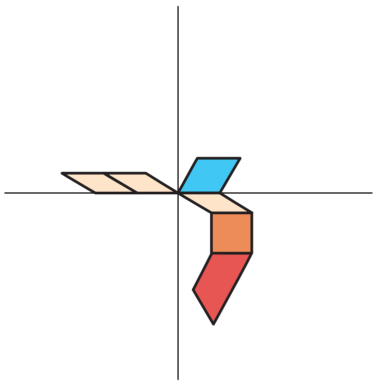
2. a. Use your pattern blocks and Material Card 33 to build this design. Complete the design by adding additional pattern blocks so that the design has reflection symmetry about both perpendicular lines. Sketch the completed design on the given axes. Does the completed design have any rotational symmetries? If so, what are they?



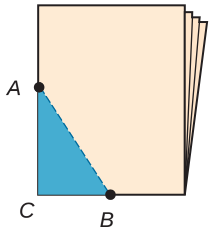


b. Build the design shown here and use additional pattern blocks to complete a design which has rotational symmetries of 90°, 180°, and 270° about the intersection of the perpendicular lines. Sketch the completed design on the given axes. Does the completed design have reflection symmetries? If so, how many? Mark any lines of symmetry on your sketch.

c. Arrange pattern blocks as shown in this figure. Add additional pattern blocks so that the completed design has rotational symmetry of 90°, 180°, and 270°. Sketch the completed design on the given axes.



Paper-Folding Symmetries

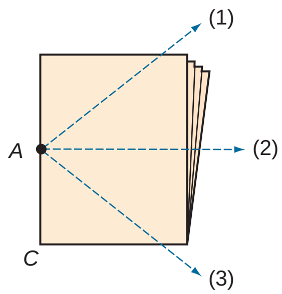


3. Fold a rectangular piece of paper in half twice, making the second fold perpendicular to the first. Let C be the corner of all folded edges. Hold the folded paper at corner C and make one cut across as shown by segment AB on the diagram. Before opening the folded corner, predict what the shape (shaded part in unfolded paper) will be and how many lines of symmetry it will have.

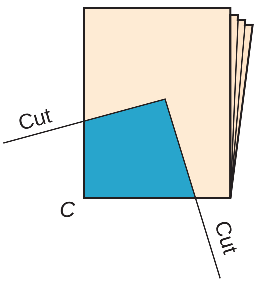
	Predicted Figure	Actual Result
Type of polygon	______________	______________
Number of lines of symmetry	______________	______________

a. Is it possible to draw a segment AB and make one cut across the folds so that the piece will unfold to a square? If so, describe how you made your cut. If not, explain why it cannot be done.

b. Using a new piece of paper each time, experiment with other single cuts that start at point A but which exit from any of the other three sides of the twice-folded paper. Continue to predict before you unfold. Make a list of the different types of figures you obtain.



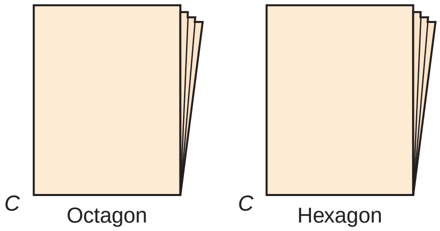
Cut	Predicted Figure	Actual Result
(1)	______________	______________
(2)	______________	______________
(3)	______________	______________



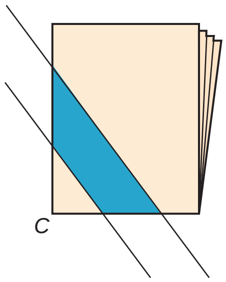
4. Fold a rectangular piece of paper in half twice, making the second fold perpendicular to the first as in activity 3. Let C be the corner of all the folded edges. Make two cuts from the edges to an inside point, as shown here. Before cutting the paper, predict what kind of polygon you will get, how many lines of symmetry the figure will have, and whether the figure will be convex or nonconvex. Cut and check your predictions.

	Predicted Figure	Actual Result
Type of polygon	______________	______________
Number of lines of symmetry	______________	______________
Convex or not	______________	______________

***a.** Find a way to make two cuts into an inside point so that you get a regular octagon; a regular hexagon. Sketch the location of your cuts on these folded-paper diagrams.

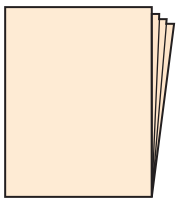


b. Make a sketch of the figure you think will result when you cut a piece of double-folded paper as shown at the left. Then cut the paper and sketch the result.



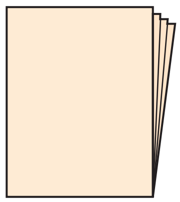
Predicted Figure	Actual Result

c. Vary the angle of the cuts in part b so that the two cuts are not parallel to each other. Experiment with cuts that will result in a rhombus inside a square; a square inside a rhombus. Sketch these lines on the folded-paper diagrams below.



Cuts for rhombus inside square

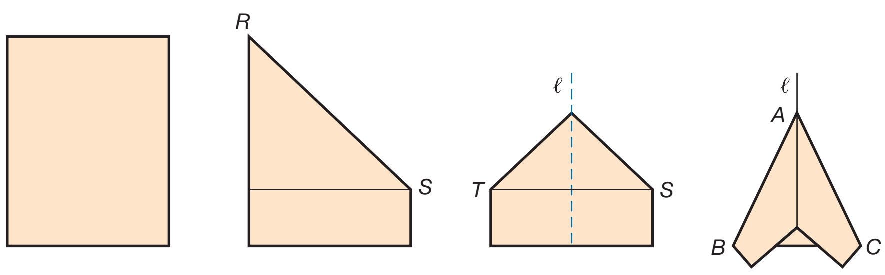
Sketch of rhombus inside square



Cuts for square inside rhombus

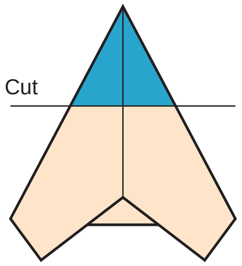
Sketch of square inside rhombus

5. The following pattern leads to a variety of symmetrical shapes. Begin with a standard sheet of paper as in figure a, and fold the upper right corner down to produce figure b. Then fold the upper vertex R down to point S to obtain figure c. To get figure d, fold the two halves inward so that points S and T are on line ℓ.



(a) (b) (c) (d)

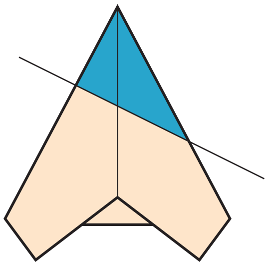
Make a horizontal cut, as indicated on this diagram, and draw a sketch of the resulting figure.



Resulting Figure

For each part a–d, draw a sketch of the figure you predict will result from the indicated cut. Then fold the paper, make the cut, and draw the actual result. Determine the number of lines of symmetry for each.

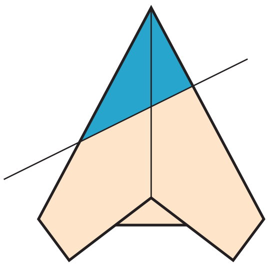
a. Slanted cut



Predicted Figure	Actual Result

Number of lines of symmetry ________

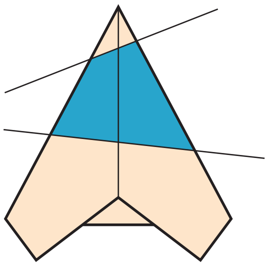
b. Slanted cut (reversed)



Predicted Figure	Actual Result

Number of lines of symmetry ________

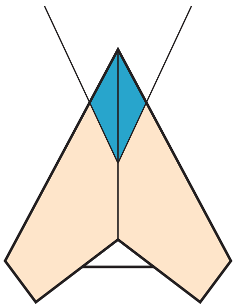
c. Combination of two slanted cuts



Predicted Figure	Actual Result

Number of lines of symmetry ________

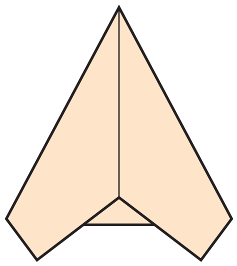
***d.** Two cuts to an inside point on the center line



Predicted Figure	Actual Result

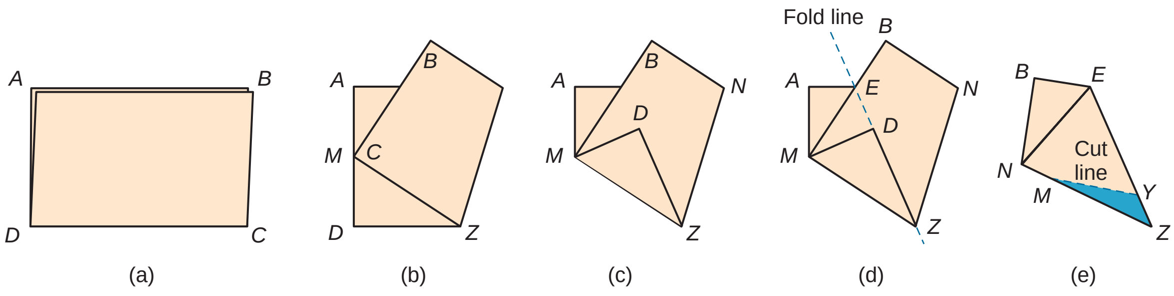
Number of lines of symmetry ________

***e.** Find two cuts from the slanted sides of the following figure into a point on the center line that will produce a regular 16-sided polygon. Sketch the cuts on this diagram.



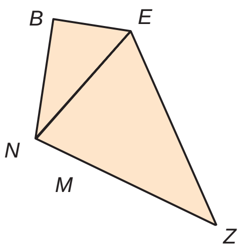
6. The five-pointed star has been used for badges and national symbols for centuries. It appears on the flags of over 40 countries and was once used on the back of a United States \$4 piece.

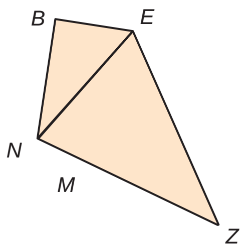
The steps pictured in figures a through d illustrate a paper-folding approach to making a five-pointed star. To obtain figure a, fold a standard sheet of *paper* perpendicular to its longer side. Next, fold point C over to midpoint M of side $\overline{AD}$, as shown in figure b. To get figure c, fold up the corner containing point D and crease along $\overline{MZ}$. The final fold line is shown in figure d. Fold the right side of figure d over to the left side so that edge NZ lies along edge MZ resulting in figure e.



a. Using your folded paper, as in figure e, make a cut from Y to M (not N) and sketch the resulting figure here. Changing the position of Y will alter the thickness of the points on your star. Hold the paper tightly when cutting so it doesn't slip.

b. Begin with a new piece of paper and use the preceding steps to obtain the pattern in figure e. Find a way to make a single cut so that the resulting figure is a regular pentagon; a regular decagon. Sketch the cut lines on the figures below.





JUST FOR FUN

SNOWFLAKES

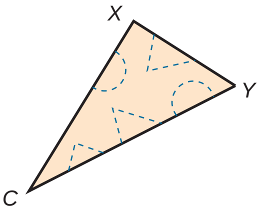
Snow is the only substance that crystallizes in many different figures. Yet, in spite of the variations in the figures, all snow crystals have a common characteristic—their hexagonal shape.

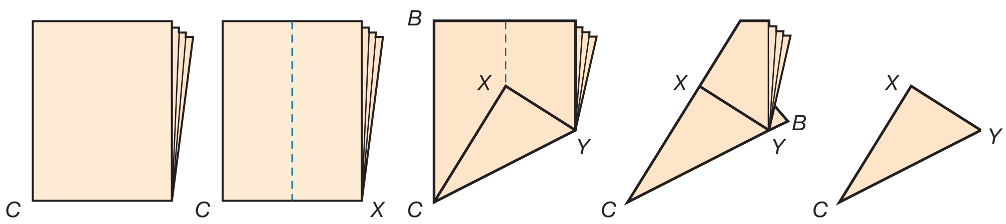
A New England farmer, W. A. Bentley, began looking at snowflakes when he was given a microscope at the age of 15. A few years later he was given a camera, which he adapted to his microscope to take photographs of snowflakes. Shortly before his death in 1931, Bentley published a book containing 2500 pictures of snow crystals. His work has been used by artists, photographers, illustrators, jewelers, meteorologists, and crystallographers.[†]

To create your own snowflake, fold a standard-size sheet of paper according to the following directions.

1. Fold the paper in half twice, making the second fold perpendicular to the first as in activity 3. In figure a the corner C is also the center of the original piece of paper.
2. Fold to find the center line parallel to the longer edges, as indicated in figure b.
3. Bring corner X up to the center line and crease the paper along CY, as shown in figure c.
4. Fold corner B back until side $\overline{CB}$ lies along side $\overline{CY}$, and crease along $\overline{CX}$, as in figure d.
5. Cut off the portion of paper above $\overline{XY}$, as shown in figure e.

Now cut out designs along sides $\overline{CY}$, $\overline{CX}$, and $\overline{XY}$. Unfold the finished product carefully to examine your snowflake design. What is its shape? What lines of symmetry and what rotational symmetries does it have? Experiment with other designs to obtain a variety of snowflakes.





(a) (b) (c) (d) (e)

[†]F. Hapgood, "When Ice Crystals Fall from the Sky, Art Meets Science," *Smithsonian,* pp. 67–72.

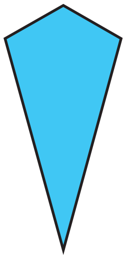
Exercises and Activities 9.4

1. *School Classroom:* One of your students claims that any line drawn through the center of a square is a line of symmetry for the square. Describe what you believe this student was thinking and how you would help her determine the lines of symmetry of a square without actually showing her these lines.

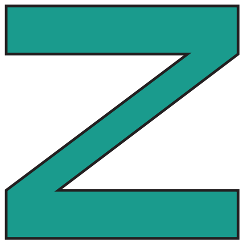
2. *School Classroom:* Rydell has found a wonderful pattern. Squares have four lines of symmetry and four rotational symmetries, nonsquare rectangles have two lines of symmetry and two rotational symmetries. He is sure this pattern (the number of lines of symmetry equals the number of rotational symmetries) holds for all quadrilaterals. Is he correct and if not, how can you help him resolve this issue?

3. *School Classroom:* Determine and describe the rotational and line symmetries for each of the following three figures.

a.



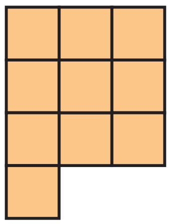
b.



c.

4. *Math Concepts:* For each of the following use any combination of Pattern Blocks (cardstock or virtual) to form the figure with the stated properties; but do not duplicate any figures formed in this activity set. Sketch or print your work. Printable Virtual Pattern Blocks are available at the companion website.
   a. Figure a: One line of symmetry and no rotational symmetries (other than 360°)
   b. Figure b: Six rotational symmetries but no lines of symmetry
   c. Figure c: Three rotational symmetries and three lines of symmetry
   d. Figure d: No rotational symmetries (other than 360°) and no lines of symmetry

5. *Math Concepts:* How many symmetrical shapes can be made by joining one more square to the ten square shape below? Use color tiles from the Manipulative Kit to find the shapes and record your answers on centimeter grid paper. Describe the type of symmetry. Centimeter Grid Paper is available for download at the companion website.



6. *Math Concepts:* Open the **Math Laboratory Investigation 9.4: Read Me—Mirror Cards Instructions** from the companion website and investigate the mirror patterns described in 1, 2, and 3 of the *Starting Points for Investigations 9.4.* Show your procedures and explain your thinking. A small handheld mirror will be helpful.

7. *NCTM Standards:* Go to http://illuminations.nctm.org/ and under "Lessons" select grade levels 3–5 and the **Geometry Standard.** Choose a lesson involving symmetry.

a. State the title of the lesson and briefly summarize the lesson.

b. Referring to the **Standards Summary** in the back pages of this book as necessary, list the *Geometry Standard Expectations* that the lesson addresses and explain how the lesson addresses these *Expectations*.

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Perpendicular Lines for Symmetry (Activity Set 9.4)

MATERIAL CARD 33