

3.2: Unit Analysis and Linear Equations

1. A person earns a starting salary of \$32 thousand dollars at a company. Each year they receive a \$2 thousand dollar raise. Let s be the person's salary, in thousands of dollars, after they have worked for the company for t years.

(a) Calculate the values of s for the given values of t and fill out the table below:

| t | s |
|-----|------------------|
| 0 | $32 + (0)2 = 32$ |
| 1 | $32 + (1)2 = 34$ |
| 2 | $32 + (2)2 = 36$ |
| 3 | $32 + (3)2 = 38$ |
| 4 | $32 + (4)2 = 40$ |

(b) What is the value of s when t is 8?

$$32 + (8)2 = 48$$

(c) What does your answer from 1b mean in this situation?

The person's salary is 48 thousand dollars after working for the company for 8 years.

(d) What is the s -intercept? What does it mean in this situation?

The s -intercept is at the point $(0, 32)$. This means that the person's starting salary was 32 thousand dollars.

(e) When will the salary be \$42 thousand dollars per year?

42 is 10 more than 32, and 10 is 2 times 5, so the salary will be 42 after 5 years.

(f) Find a linear equation of the form $s = mt + b$ that models this data.

The s -intercept is 32 and the slope is 2 so the equation is

$$s = 32 + 2t.$$

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2. Rewrite the equation that you found in problem 1f: $s = 32 + 2t$. In this equation the variable s is given in thousands of dollars, and t is given in years. These are called the **units** of the variable. In any equation the units on the left side must be equal to the units on the right side.

(a) In the equation you found in problem 1f, what is b , and what does it mean in this situation? What are the units of b .

b is the s -intercept, which in this case is 32. This means that the starting salary is 32 thousand dollars. The units of b are thousands of dollars.

(b) What is m , and what does it mean in this situation? What are the units of m ?

m is the slope, in this case it is 2. In this case it means that the salary is increasing by 2 thousand dollars per year. The units of m are thousands of dollars per year.

(c) Why are the units of m different from the units of b .

The units of m are different than those of b because m represents how much a quantity is changing with respect to time, while b represents a fixed starting income.

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3. Between the years of 2009 and 2013 the minimum wage remained constant at \$7.25 per hour. Let w be the minimum wage, in dollars, at t years after 2009.

(a) Use the above information to fill out the table:

| t | w |
|---|----------------------|
| 0 | 7.25 |
| 1 | $7.25 + 0(1) = 7.25$ |
| 2 | $7.25 + 0(2) = 7.25$ |
| 3 | $7.25 + 0(3) = 7.25$ |
| 4 | $7.25 + 0(4) = 7.25$ |

(b) What are the units of w ? What are the units of t ?

The units of w are dollars per hour. The units of t is years.

(c) Find a linear equation of the form $w = mt + b$ that models this data.

We need to find m and b ; b is value of w at time $t = 0$. In other word b is the value when we started, which is 7.25. The value of m is determined by how much w changes from year to year. In this case w does not change from year to year. So we have that $m = 0$. Thus a linear equation that models this data is

$$y = 0t + 7.25 = 7.25$$

(d) What is b , and what does it mean in this case? What units does b have?

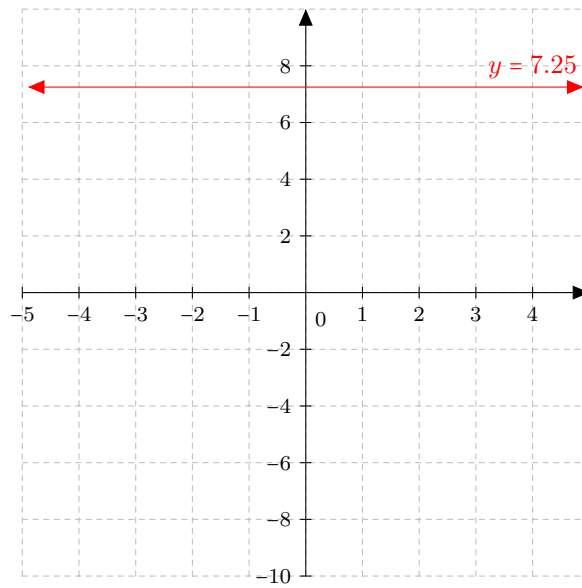
In this case $b = 7.25$. This represents the minimum wage when $t = 0$, which is in 2009. The units of b are given in dollars per hour.

(e) What is m , and what does it mean in this case? What units does m have?

Here m represents the slope or the rate of change of w with respect to time. In this case w is not changing with time, so $m = 0$. Even though $m = 0$ it still has units of dollars per hour per year.

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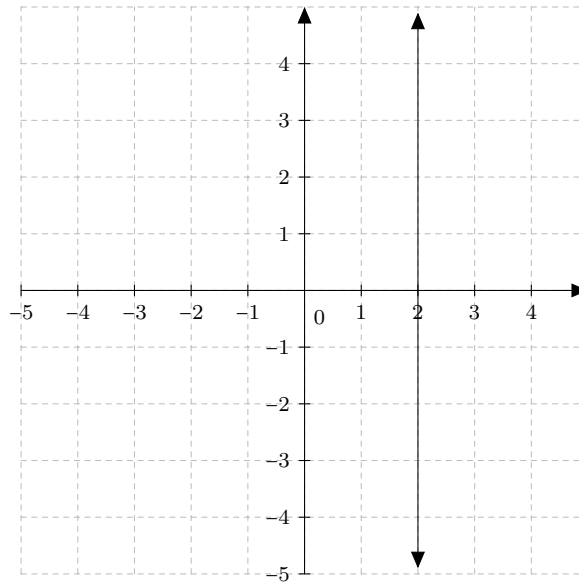
- (f) Rewrite the equation that you found in problem 3c: $y=7.25$. Graph this equation.



- (g) For a linear equation $y = mx + b$, the number m represents the rate of change of the quantity y . In our current situation we find that $m = 0$. What is the slope of the graph you made above?

The slope of the above line is 0.

4. In question 3 we found that the data was modeled by a horizontal line. This meant that the rate of change of the variable w was zero, and therefore it was constant. Consider the following graph:



- (a) List four points that are on this graph:

$(2, 0)$, $(2, -1)$, $(2, 1)$, and $(2, 2)$. Answers may vary, but any point of the form $(2, y)$ is on the line.

- (b) The slope is thought of as change in y divided by change in x . Choose two of the points that are on this graph. What is the change in y ? What is the change in x ? Can you calculate the slope of this line? Why or Why not?

Let us use the points $(2, 0)$ and $(2, 1)$. The change in y between the first and the second is $1 - 0 = 1$. The change in x between the first and the second is $2 - 2 = 0$. I cannot calculate the slope of this line. The slope of this line should be the quotient of 1 divided by 0, but this does not make sense. Therefore the slope of this line is not well defined.

- (c) What is an equation of this line?

We can see that every point on the line satisfies the equations $x = 2$. Furthermore any point that satisfies the equation $x = 2$ is on the line. Therefore we can conclude that the equation of this line is

$$x = 2$$

- (d) Can this be written in the form of $y = mx + b$. Why or Why not? What does this mean? Use the back of this page if you need more room to write.

Because the slope of this line is not well defined, we cannot use the equation $y = mx + b$, since it is an expression involving the slope. This means that not every straight line can be represented by an equation $y = mx + b$, only those that are non-vertical have this form, and vertical lines have the form $x = c$ for some number c .